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Determining F_π from spectral sum rules

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Abstract

We derive spectral sum rules for a system with two quarks coupled to an imaginary isospin chemical potential in the ϵ regime. The sum rules show an explicit dependence on the pion decay constant which should make it possible to measure F_π from the eigenvalue spectrum of this particular Dirac operator.

1. The determination of low energy constants of QCD such as the pion decay constant F_π remains an important problem. In particular the computation of such quantities from lattice calculations is notoriously difficult due to the exceeding computational challenge posed by simulations of small quark masses. In addition when approaching the chiral limit on the lattice, finite size effects inevitably become more and more significant. Hence, a method which takes finite size scaling explicitly into account seems to be highly profitable. Such a technique is provided by the so-called ϵ-regime of QCD [1]. This regime applies in a region where the Compton wave length of the pion $1/m_\pi$ is larger than the one-dimensional size L of the physical volume $V = L^4$, while still being much smaller than the typical hadronic scale $\Lambda_{\rm QCD}$, i.e. $1/m_\pi > L \gg 1/\Lambda_{\rm QCD}$. The lowest order effective partition function of the ϵ-regime is known analytically. The fact that it depends explicitly on the infinite volume chiral condensate $\Sigma = \langle\bar\psi\psi\rangle$ can be exploited to determine this constant with high accuracy: back in 1992 Leutwyler and Smilga derived a set of spectral sum rules for the Dirac operator by restricting the partition function to sectors with fixed topological charge ν [2]. In this way the chiral condensate is linked to the spectrum of the Dirac operator in finite volume which can be determined from lattice simulations [3,4].

However, with the standard Dirac operator other low energy constants such as the pion decay constant F_π appear only in higher order corrections [5]. As a consequence, the computation of F_π was believed to be much more demanding [4,6,7]. It is therefore still more widespread to extrapolate the pion decay constant down to the chiral limit from simulations in the p regime of chiral perturbation theory [8,9].

Recently a new approach has been proposed [10–12] which avoids these difficulties: if the quarks are coupled to an external source, dependency on F_π appears already at the lowest order in the partition function [5,13]. The authors of Refs. [10,11] have used this fact to derive correlation functions of the eigenvalue densities which are sensitive to F_π for both quenched and unquenched chiral perturbation theory. Here, we will use the same partition function to deduce a set of sum rules in a way analogous to Ref. [2]. These rules depend then likewise on F_π and make it possible to determine this important quantity from a lattice simulations in finite volume.

2. We consider a system with two quark flavors u and d which are coupled to an external source $\mu_{\rm iso}$. The source can be interpreted as an imaginary isospin chemical potential which couples differently to the two flavors or as twisted boundary conditions

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[12,14]. This gives rise to two independent eigenvalue equations

$$(D_+ + m_u)\psi_{+n} = \big(\not{D}[A] + i\mu_{\rm iso}\gamma_0 + m_u\big)\psi_{+n} = (i\lambda_{+n} + m_u)\psi_{+n}, \tag{1}$$

$$(D_- + m_d)\psi_{-n} = \big(\not{D}[A] - i\mu_{\rm iso}\gamma_0 + m_d\big)\psi_{-n} = (i\lambda_{-n} + m_d)\psi_{-n}, \tag{2}$$

where A denotes the gauge field. The advantage of an *imaginary isospin* chemical potential is twofold. Firstly, an isospin chemical potential preserves the positivity of the fermion matrix [14]. The system can thus be simulated on the lattice with the usual Monte Carlo techniques without running into sign problems. Secondly, if it is chosen to be imaginary the massless operators D_+ and D_- are anti-Hermitian and the eigenvalues $\lambda_\pm$ lie on the real axis.

In the case of degenerate masses $m_u = m_d$ the two flavors can be converted into each other by the transformation $\mu_{\rm iso}$ to $-\mu_{\rm iso}$. Here we use a non-mass-degenerate quark pair for purely technical reasons: as we will see below this gives us the only handle to distinguish between the two sets of eigenvalues. Thus keeping the masses distinct allows us to extract sum rules for only one set of eigenvalues, λ_+ or λ_- accordingly.

For both sets of eigenvalues the non-zero modes come in positive and negative pairs, $\pm\lambda_{+n}$ and $\pm\lambda_{-n}$, respectively. Hence in a sector of fixed topology, the partition function can be written as

$$Z_\nu(m_u, m_d) = \left\langle m_u^\nu m_d^\nu \prod_n \big(\lambda_{+n}^2 + m_u^2\big) \prod_n \big(\lambda_{-n}^2 + m_d^2\big) \right\rangle_\nu, \tag{3}$$

where $\langle\cdot\rangle_\nu$ denotes the gauge average over all configurations with topological charge ν and the products are restricted to strictly positive values of λ.

Our derivation of the sum rules deviates from the procedure originally used by Leutwyler and Smilga in Ref. [2], we will follow the somewhat easier approach presented in Ref. [15] instead. Massive spectral sum rules for λ_+ (or λ_-) can be derived from formula (3) by taking logarithmic derivatives with respect to the masses m_u (or m_d). A first order sum rule for instance is given by

$$\frac{1}{2m_u}\left(\frac{\partial}{\partial m_u}\ln Z_\nu(m_u, m_d) - \frac{\nu}{m_u}\right) = \left\langle \sum_n \frac{1}{\lambda_{+n}^2 + m_u^2} \right\rangle_\nu. \tag{4}$$

As in the products of Eq. (3) the sum on the rhs runs over the positive eigenvalues only.

On the other hand in the ϵ expansion of chiral perturbation theory the partition function can be shown [11,13] to be

$$Z_\nu(x_u, x_d) = \mathrm{e}^{-2VF_\pi^2\mu_{\rm iso}^2} \int_0^1 \mathrm{d}s\, \mathrm{e}^{2VF_\pi^2\mu_{\rm iso}^2 s^2}\, s\, I_\nu(sx_d) I_\nu(sx_u), \tag{5}$$

where we introduced the scaling variables of the ϵ-regime $x_i = V\Sigma m_i$, $i = u, d$ and I_ν are modified Bessel functions. The dependence on F_π is through the product $VF_\pi^2\mu_{\rm iso}^2$ only. In particular a sign change in $\mu_{\rm iso}$ leaves the partition function invariant. The two flavors are indeed only distinguishable through their masses. Upon inserting this explicit formula for Z_ν into Eq. (4), we obtain the first order sum rule

$$\left\langle \sum_n \frac{1}{\lambda_{+n}^2 + m_u^2} \right\rangle_\nu = \frac{V^2\Sigma^2}{2x_u} \frac{\int_0^1 \mathrm{d}s\, \mathrm{e}^{2VF_\pi^2\mu_{\rm iso}^2 s^2} s^2 I_\nu(sx_d) I_{\nu+1}(sx_u)}{\int_0^1 \mathrm{d}s\, \mathrm{e}^{2VF_\pi^2\mu_{\rm iso}^2 s^2} s\, I_\nu(sx_d) I_\nu(sx_u)}. \tag{6}$$

Eq. (6) has to be taken with some care. The lhs needs to be properly regularized. That this expression is UV finite can be seen by the following argument [2]: the lhs of Eq. (6) is actually proportional to the quark condensate $\langle\bar{u}u\rangle$

$$\langle\bar{u}u\rangle = -\lim_{V\to\infty} \frac{2m_u}{V} \sum_\nu \left\langle \sum_n \frac{1}{\lambda_{+n}^2 + m_u^2} \right\rangle_\nu = -2m_u \int_0^\infty \mathrm{d}\lambda_+ \frac{\rho(\lambda_+)}{\lambda_+^2 + m_u^2}, \tag{7}$$

as the eigenvalues become dense in the infinite volume limit. Eq. (7) is ultraviolet divergent, because the density scales proportional to λ_+^3 for large λ_+. It can however be regularized by the introduction of counterterms of order m_u and m_u^3 and a proper tuning of their coefficients. This argument is related to the fact that the partition function of QCD can be made finite by the introduction of a cosmological constant. As a consequence, higher order sum rules can be shown to be finite by the same argument. Let us introduce a cutoff Λ such that $m_u \ll \Lambda$ and add the counterterms

$$\langle\bar{u}u\rangle = -2m_u \int_0^\Lambda \mathrm{d}\lambda_+ \frac{\rho(\lambda_+)}{m_u^2 + \lambda_+^2} - 2m_u \int_\Lambda^\infty \mathrm{d}\lambda_+ \frac{\rho(\lambda_+)}{m_u^2 + \lambda_+^2} + c_1 m_u + c_2 m_u^3. \tag{8}$$

The divergent contributions can be subtracted from the high momentum part of the integral

$$2m_u\int_{\Lambda}^{\infty}\mathrm{d}\lambda_+\frac{\rho(\lambda_+)}{m_u^2+\lambda_+^2}+c_1m_u+c_2m_u^3=2m_u^5\int_{\Lambda}^{\infty}\mathrm{d}\lambda_+\frac{\rho(\lambda_+)}{\lambda^4(m_u^2+\lambda_+^2)}+\gamma_1m_u+\gamma_2m_u^3. \tag{9}$$

All these terms disappear, as the mass is taken to zero and we are left with the low momenta contribution in Eq. (8). In lattice simulations we have of course a finite mass and a finite volume, but as we keep the scaling variable $x_i = V\Sigma m_i$ constant and simulate at a fixed cutoff the correction terms become irrelevant as we go to larger lattices.

Formula (6) simplifies a good deal if we take both quark masses to zero. The Bessel functions disappear up to remnant powers of the parameter s

$$\left\langle\sum_n\frac{1}{\lambda_{+n}^2}\right\rangle_\nu=\frac{V^2\Sigma^2}{4(\nu+1)}\frac{\int_0^1\mathrm{d}s\,\mathrm{e}^{2VF_\pi^2\mu_{\mathrm{iso}}^2s^2}s^{2\nu+3}}{\int_0^1\mathrm{d}s\,\mathrm{e}^{2VF_\pi^2\mu_{\mathrm{iso}}^2s^2}s^{2\nu+1}}. \tag{10}$$

For general ν this expression can only be evaluated in terms of incomplete and ordinary Γ-functions

$$\left\langle\sum_n\frac{1}{\lambda_{+n}^2}\right\rangle_\nu=-\frac{V\Sigma^2}{8(\nu+1)F_\pi^2\mu_{\mathrm{iso}}^2}\frac{\Gamma(\nu+2)-\Gamma(\nu+2,-2VF_\pi^2\mu_{\mathrm{iso}}^2)}{\Gamma(\nu+1)-\Gamma(\nu+1,-2VF_\pi^2\mu_{\mathrm{iso}}^2)}=-\frac{V\Sigma^2}{8(\nu+1)F_\pi^2\mu_{\mathrm{iso}}^2}\frac{\int_{-2VF_\pi^2\mu_{\mathrm{iso}}^2}^0t^{\nu+1}\mathrm{e}^{-t}\mathrm{d}t}{\int_{-2VF_\pi^2\mu_{\mathrm{iso}}^2}^0t^{\nu}\mathrm{e}^{-t}\mathrm{d}t}. \tag{11}$$

However, for a given topological charge it is straightforward to calculate the parameter integrals. The rule reduces then to a rather simple expression, where F_π appears only in polynomials and exponentials and which can easily be fitted to lattice data. Let us illustrate this for the case of vanishing topological charge, there we simply have

$$\left\langle\sum_n\frac{1}{\lambda_{+n}^2}\right\rangle_0=\frac{V\Sigma^2}{8F_\pi^2\mu_{\mathrm{iso}}^2}\frac{1+\mathrm{e}^{2VF_\pi^2\mu_{\mathrm{iso}}^2}(2VF_\pi^2\mu_{\mathrm{iso}}^2-1)}{\mathrm{e}^{2VF_\pi^2\mu_{\mathrm{iso}}^2}-1}. \tag{12}$$

Eqs. (10) and (11) are the direct equivalents of the first order sum rule given in Ref. [2], but evaluated for a system with two quarks coupled the chemical potential μ_{iso}. Indeed, in the limit where μ_{iso} vanishes Eq. (10) becomes

$$\left\langle\sum_n\frac{1}{\lambda_{+n}^2}\right\rangle_\nu=\frac{V^2\Sigma^2}{4(\nu+1)}\frac{\int_0^1\mathrm{d}s\,s^{2\nu+3}}{\int_0^1\mathrm{d}s\,s^{2\nu+1}}=\frac{V^2\Sigma^2}{4(\nu+2)}, \tag{13}$$

which is precisely the Leutwyler–Smilga result for $N_f = 2$ flavors.

Due to the symmetry of the partition function under an exchange of the quark masses, the corresponding sum rule for λ_- can be obtained from Eq. (6) by simply substituting x_u for x_d. If we had treated the quarks as mass degenerate from the beginning we would have arrived at the sum of those two sum rules, which is just twice Eq. (6).

Higher order sum rules

We can derive two different types of second order sum rules by either taking the second derivative with respect to m_u, $\frac{\partial^2}{\partial m_u^2}\ln Z_\nu$, or a mixed derivative $\frac{\partial^2}{\partial m_u\partial m_d}\ln Z_\nu$. Let us start with the second possibility. Applied to Eq. (3) the mixed derivative yields the subtracted correlation of the two sets of eigenvalues

$$\frac{1}{4m_um_d}\left[\frac{\partial^2}{\partial m_u\partial m_d}\ln Z_\nu(m_u,m_d)\right]=\left\langle\left(\sum_n\frac{1}{\lambda_{+n}^2+m_u^2}\right)\left(\sum_n\frac{1}{\lambda_{-n}^2+m_d^2}\right)\right\rangle_\nu-\left\langle\sum_n\frac{1}{\lambda_{+n}^2+m_u^2}\right\rangle_\nu\left\langle\sum_n\frac{1}{\lambda_{-n}^2+m_d^2}\right\rangle_\nu. \tag{14}$$

Since we have calculated the disconnected parts already, the only new contribution from Eq. (14) is a sum rule for

$$\left\langle\left(\sum_n\frac{1}{\lambda_{+n}^2+m_u^2}\right)\left(\sum_n\frac{1}{\lambda_{-n}^2+m_d^2}\right)\right\rangle_\nu=\frac{V^4\Sigma^4}{4x_ux_d}\frac{\int_0^1\mathrm{d}s\,\mathrm{e}^{2VF_\pi^2\mu_{\mathrm{iso}}^2s^2}s^3I_{\nu+1}(sx_u)I_{\nu+1}(sx_d)}{\int_0^1\mathrm{d}s\,\mathrm{e}^{2VF_\pi^2\mu_{\mathrm{iso}}^2s^2}sI_\nu(sx_u)I_\nu(sx_d)}. \tag{15}$$

In the limit of vanishing quark masses we can again expand the Bessel functions and obtain a much simpler expression for the massless mixed second order sum rule

$$\begin{aligned}\left\langle\left(\sum_n\frac{1}{\lambda_{+n}^2}\right)\left(\sum_n\frac{1}{\lambda_{-n}^2}\right)\right\rangle_\nu&=\frac{V^4\Sigma^4}{16(\nu+1)^2}\frac{\int_0^1\mathrm{d}s\,\mathrm{e}^{2VF_\pi^2\mu_{\mathrm{iso}}^2s^2}s^{2\nu+5}}{\int_0^1\mathrm{d}s\,\mathrm{e}^{2VF_\pi^2\mu_{\mathrm{iso}}^2s^2}s^{2\nu+1}}\\&=\frac{V^2\Sigma^4}{64F_\pi^4\mu_{\mathrm{iso}}^4(\nu+1)^2}\frac{\Gamma(\nu+3)-\Gamma(\nu+3,-2VF_\pi^2\mu_{\mathrm{iso}}^2)}{\Gamma(\nu+1)-\Gamma(\nu+1,-2VF_\pi^2\mu_{\mathrm{iso}}^2)}.\end{aligned} \tag{16}$$

Apart from the coefficients, the difference to the massless first order rule is given by the higher power of the parameter s in the first line of Eq. (16). These powers are indeed a distinctive feature for any massless sum rule of a given order (third order sum rules for instance carry a power of $s^{2\nu+7}$). Concerning the evaluation of the integral we can make exactly the same remarks as for the first order rule, they are easily calculated in a fixed topological sector. For completeness we give again the result at $\nu = 0$

$$\left\langle \left(\sum_n \frac{1}{\lambda_{+n}^2}\right)\left(\sum_n \frac{1}{\lambda_{-n}^2}\right)\right\rangle_0 = \frac{V^2\Sigma^4}{32F_\pi^4\mu_{\rm iso}^4}\,\frac{(1-2VF_\pi^2\mu_{\rm iso}^2(1+VF_\pi^2\mu_{\rm iso}^2))\mathrm{e}^{2VF_\pi^2\mu_{\rm iso}^2}-1}{\mathrm{e}^{2VF_\pi^2\mu_{\rm iso}^2}-1}. \tag{17}$$

Again, in the limit $\mu_{\rm iso} \to 0$ where λ_+ and λ_- become degenerate the formula reproduces the result of Ref. [2]. In addition, this second order sum rule can be compared to the results of Ref. [11]. There the mixed two point spectral correlation function

$$\rho^{(2)}(\lambda_1,\lambda_2,m_u,m_d,i\mu_{\rm iso}) = \left\langle \sum_n \delta(\lambda_1-\lambda_{+n}) \sum_l \delta(\lambda_2-\lambda_{-l})\right\rangle - \left\langle \sum_n \delta(\lambda_1-\lambda_{+n})\right\rangle\left\langle \sum_l \delta(\lambda_2-\lambda_{+n})\right\rangle \tag{18}$$

is derived with the replica method. By integrating out λ_1 and λ_2 in

$$\int \mathrm{d}\lambda_1 \int \mathrm{d}\lambda_2\, \frac{\rho^{(2)}(\lambda_1,\lambda_2,m_u,m_d,i\mu_{\rm iso})}{\lambda_1^2\lambda_2^2} \tag{19}$$

we should reproduce our subtracted mixed sum rule of Eq. (14). Indeed, we do find exact agreement in the massless limit.

With the first choice above $\frac{\partial^2}{\partial m_u^2}\ln Z_\nu$, we have means to extract the correlation between eigenvalues of one series λ_+ (or equivalently λ_-). More precisely, from taking the second derivative with respect to m_u and subtracting the zero mode contributions and the known first order terms, we can deduce a formula for

$$\left\langle \left(\sum_n \frac{1}{\lambda_{+n}^2+m_u^2}\right)^2 - \sum_n \left(\frac{1}{\lambda_{+n}^2+m_u^2}\right)^2\right\rangle_\nu = \frac{V^4\Sigma^4}{4x_u^3}\,\frac{\int_0^1 \mathrm{d}s\,\mathrm{e}^{2VF_\pi^2\mu_{\rm iso}^2 s^2} s^2 I_\nu(sx_d)[sx_u I_\nu(sx_u) - 2(\nu+1)I_{\nu+1}(sx_u)]}{\int_0^1 \mathrm{d}s\,\mathrm{e}^{2VF_\pi^2\mu_{\rm iso}^2 s^2} s I_\nu(sx_d) I_\nu(sx_u)}. \tag{20}$$

Again the formula simplifies drastically in the massless limit where we get

$$\left\langle \left(\sum_n \frac{1}{\lambda_{+n}^2}\right)^2 - \sum_n \frac{1}{\lambda_{+n}^4}\right\rangle_\nu = \frac{V^4\Sigma^4}{16(\nu+1)(\nu+2)}\,\frac{\int_0^1 \mathrm{d}s\,\mathrm{e}^{2VF_\pi^2\mu_{\rm iso}^2 s^2} s^{2\nu+5}}{\int_0^1 \mathrm{d}s\,\mathrm{e}^{2VF_\pi^2\mu_{\rm iso}^2 s^2} s^{2\nu+1}}. \tag{21}$$

Let us remark here that at finite $\mu_{\rm iso}$ we cannot extract a separate sum rule for the fourth order term $\langle\sum 1/\lambda_+^4\rangle_\nu$ from the partition function in Eq. (5). Since the u and the d quark obey two different eigenvalue equations, each of them behaves effectively like a one flavor system. In order to isolate the fourth order term above we would need another flavor with the same eigenvalues λ_{+n} but different mass $\tilde{m}_u$. At vanishing isospin chemical potential however, where λ_+ and λ_- become degenerate a fourth order term can be calculated from the difference of the two second order sum rules described here and it again coincides with the result of Ref. [2].

It is in principle possible to continue this series and to calculate sum rules of any order by taking higher and higher derivatives, and subtracting the known lower order terms. The computation however becomes tedious and its usefulness doubtful.

3. In this Letter we have derived a set of spectral sum rules for a system of two quarks coupled to an imaginary isospin chemical potential from the finite volume partition function. This can be seen as an extension of the pioneering work of Leutwyler and Smilga in Ref. [2]. The sum rules derived here inherit the F_π dependence from the partition function and provide means to determine this important constant from the spectrum of the Dirac operator introduced here.

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References

[1] J. Gasser, H. Leutwyler, Phys. Lett. B 188 (1987) 477.
[2] H. Leutwyler, A. Smilga, Phys. Rev. D 46 (1992) 5607.
[3] E.V. Shuryak, J.J.M. Verbaarschot, Nucl. Phys. A 560 (1993) 306, hep-th/9212088;
M.E. Berbenni-Bitsch, S. Meyer, A. Schafer, J.J.M. Verbaarschot, T. Wettig, Phys. Rev. Lett. 80 (1998) 1146, hep-lat/9704018;
M.E. Berbenni-Bitsch, S. Meyer, T. Wettig, Phys. Rev. D 58 (1998) 071502, hep-lat/9804030;
P.H. Damgaard, U.M. Heller, A. Krasnitz, Phys. Lett. B 445 (1999) 366, hep-lat/9810060;
R.G. Edwards, U.M. Heller, R. Narayanan, Phys. Rev. D 60 (1999) 077502, hep-lat/9902021;
L. Giusti, M. Lüscher, P. Weisz, H. Wittig, JHEP 0311 (2003) 023, hep-lat/0309189.

[4] K. Ogawa, S. Hashimoto, Prog. Theor. Phys. 114 (2005) 609, hep-lat/0505017.
[5] P.H. Damgaard, M.C. Diamantini, P. Hernandez, K. Jansen, Nucl. Phys. B 629 (2002) 445, hep-lat/0112016;
P.H. Damgaard, Nucl. Phys. B 608 (2001) 162, hep-lat/0105010.
[6] W. Bietenholz, T. Chiarappa, K. Jansen, K.I. Nagai, S. Shcheredin, JHEP 0402 (2004) 023, hep-lat/0311012.
[7] L. Giusti, P. Hernandez, M. Laine, P. Weisz, H. Wittig, JHEP 0404 (2004) 013, hep-lat/0402002.
[8] S. Aoki, et al., JLQCD Collaboration, Phys. Rev. D 62 (2000) 094501, hep-lat/9912007;
S.J. Dong, T. Draper, I. Horvath, F.X. Lee, K.F. Liu, J.B. Zhang, Phys. Rev. D 65 (2002) 054507, hep-lat/0108020;
T.W. Chiu, T.H. Hsieh, Nucl. Phys. B 673 (2003) 217, hep-lat/0305016;
S. Dürr, C. Hoelbling, Phys. Rev. D 72 (2005) 071501, hep-ph/0508085;
C. Gattringer, et al., BGR Collaboration, Phys. Rev. D 72 (2005) 094510, hep-lat/0509003.
[9] C.R. Allton, et al., UKQCD Collaboration, Phys. Rev. D 70 (2004) 014501, hep-lat/0403007;
F. Farchioni, et al., $qq+q$ Collaboration, Eur. Phys. J. C 37 (2004) 197, hep-lat/0403014;
C. Aubin, et al., MILC Collaboration, Phys. Rev. D 70 (2004) 114501, hep-lat/0407028;
M.F. Lin, PoS LAT2005 (2006) 094, hep-lat/0509178.
[10] P.H. Damgaard, U.M. Heller, K. Splittorff, B. Svetitsky, D. Toublan, Phys. Rev. D 73 (2006) 074023, hep-lat/0602030.
[11] P.H. Damgaard, U.M. Heller, K. Splittorff, B. Svetitsky, Phys. Rev. D 72 (2005) 091501, hep-lat/0508029.
[12] T. Mehen, B.C. Tiburzi, Phys. Rev. D 72 (2005) 014501, hep-lat/0505014.
[13] K. Splittorff, J.J.M. Verbaarschot, Nucl. Phys. B 683 (2004) 467, hep-th/0310271;
G. Akemann, Y.V. Fyodorov, G. Vernizzi, Nucl. Phys. B 694 (2004) 59, hep-th/0404063.
[14] D.T. Son, M.A. Stephanov, Phys. Rev. Lett. 86 (2001) 592, hep-ph/0005225.
[15] P.H. Damgaard, Phys. Lett. B 425 (1998) 151, hep-th/9711047.